2 Coordinate geometry

Straight Line

given two points $A(x_1, y_1)$ *and* $B(x_2, y_2)$

Distance between two points: $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Midpoint of two points: $M_{AB} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

Gradient of two points: $m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = tan\theta$

Equation of line through a point: $y = mx + c$

Equation of horizontal line: $y = k$

Equation of vertical line: $x = k$

Given 2 straight lines

$y = m_1x + c_1$ and $y = m_2x + c_2$

then the lines are parallel if $m_1 = m_2$

and perpendicular if $m_1 \times m_2 = -1$

$2y + 6x + 5 = 0$ is parallel to $y + 3x + 4 = 0$

(because when rearranged to be $y = mx + c$ the x co-ord is -3 in both)

$2y - x + 3 = 0$ is perpendicular to $y + 2x - 4 = 0$

(when rearranged to $y = mx + c$ the x co-ords multiply to -1)

Parallel and perpendicular lines

Two lines are parallel if they have the same gradient
and they are perpendicular if the product of their gradients is –1.

Example 1:

The coordinates of the points *F* and *G* are (–2, 14) and (4, 6) respectively.

Find the equation of the straight line that

- passes through the mid-point of the line *FG*, and
- is perpendicular to the line *FG*.

Express your answer in the form $ax + by + c = 0$, where a, b and c are integers.

Gradient of FG = $\frac{6-14}{4-(-2)} = \frac{-8}{6} = -\frac{4}{3}$ Gradient of perpendicular = $\frac{3}{4}$

Midpoint of FG = $\left(\frac{-2+4}{2}, \frac{14+6}{2}\right) = (1,10)$

Equation of perpendicular is $y = mx + c$ $m = \frac{3}{4}$ and perpendicular goes through (1,10)

So $10 = \frac{3}{4}.1 + c$ $c = 10 - \frac{3}{4} = \frac{37}{4}$

Therefore equation of perpendicular is $y = \frac{3}{4}x + \frac{37}{4}$

$$\mathbf{3x - 4y + 37 = 0}$$

Example 2: Find the equation of the line through (4½, 1) and perpendicular to the line joining the points *A* (3, 7) and *B* (6, -5).

Solution: Gradient of *AB* is $\frac{-5-7}{6-3} = \frac{-12}{3} = -4$

$\Rightarrow$ gradient of line perpendicular to *AB* is ¼ , (product of perpendicular gradients is –1) so we want the line through (4½, 1) with gradient ¼ .

Using $y = mx + c \Rightarrow$ $y = ¼x + c$ $\Rightarrow 1 = \frac{1}{4}.\frac{9}{2} + c$ $\Rightarrow c = 1 - \frac{9}{8} = -\frac{1}{8}$

$\Rightarrow$ $y = \frac{1}{4}x - \frac{1}{8}$ or $-2x + 8y + 1 = 0$.