

Edexcel GCSE

Mathematics (Linear)

A* Paper (not for the faint hearted)

Higher Tier

Time: 2 hours

Materials required for examination

Ruler graduated in centimetres and millimetres, protractor, compasses, pen, HB pencil, eraser.
Tracing paper may be used.

Items included with question papers

Nil

Instructions to Candidates

Check that you have the correct brain power required to attempt this question paper.

Answer ALL the questions. Write your answers in the spaces provided in this question paper.

You must NOT phone a friend or ask the audience.

Anything you write on the formulae page will gain NO credit.

If you need more space to complete your answer to any question, write smaller.

Information for Candidates

The marks for individual questions and the parts of questions are shown in round brackets: e.g. (2).

There are 23 questions in this question paper. The total mark for this paper is 110.

Calculators must not be used unless the  symbol appears

Advice to Candidates

Show all stages in any calculations – A* questions often require you to **explain** or **prove** something.

Work steadily through the paper. Do not spend too long on one question.

If you cannot answer a question, leave it, attempt the next one and try not to cry.

Return at the end to those you have left out.

Have a lie down afterwards to help recover.

GCSE A* Questions

Skill: Manipulate expressions containing surds

Question 1

(a) Rationalise $\frac{1}{\sqrt{7}}$

.....

(2)

(b)(i) Expand and simplify

$$(\sqrt{3} + \sqrt{15})^2$$

Give your answer in the form $n + m\sqrt{5}$, where n and m are integers.

.....

(ii)

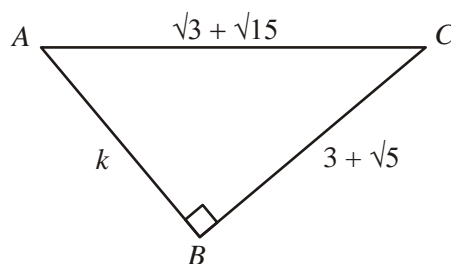


Diagram **NOT**
accurately drawn

ABC is a right-angled triangle. k is a positive integer.

Find the value of k .

$$k = \dots\dots\dots$$

(5)

(Total 7 marks)

GCSE A* Questions

Skill: Solve direct and inverse variation problems

Question 2

The force, F , between two magnets is inversely proportional to the square of the distance, x , between them.

When $x = 3$, $F = 4$.

(a) Find an expression for F in terms of x .

$$F = \dots\dots\dots \quad (3)$$

(b) Calculate F when $x = 2$.

$$\dots\dots\dots \quad (1)$$

(c) Calculate x when $F = 64$.

$$\dots\dots\dots \quad (2)$$

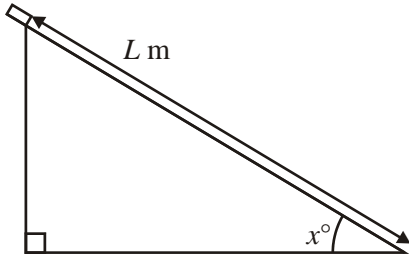
(Total 6 marks)

GCSE A* Questions



Skill: Calculate the limits of compound measures

Question 3



Elliot did an experiment to find the value of $g \text{ m/s}^2$, the acceleration due to gravity.

He measured the time, T seconds, that a block took to slide L m down a smooth slope of angle x° .

He then used the formula $g = \frac{2L}{T^2 \sin x^\circ}$ to calculate an estimate for g .

$T = 1.3$ correct to 1 decimal place. $L = 4.50$ correct to 2 decimal places. $x = 30$ correct to the nearest integer.

- (a) Calculate the lower bound and the upper bound for the value of g . Give your answers correct to 3 decimal places.

Lower bound

Upper bound

(4)

- (b) Use your answers to part (a) to write down the value of g to a suitable degree of accuracy. Explain your reasoning.

.....

(1)

(Total 5 marks)

GCSE A* Questions

Skill: Solve a pair of simultaneous equations where one is linear and the other is non-linear

Question 4

Solve the simultaneous equations

$$x^2 + y^2 = 29$$

$$y - x = 3$$

.....
(Total 7 marks)

GCSE A* Questions

Skill: Rearrange more complicated formulae where the subject may appear twice or as a power

Question 5

$$P = \frac{n^2 + a}{n + a}$$

Rearrange the formula to make a the subject.

$$a = \dots\dots\dots$$

(Total 4 marks)

GCSE A* Questions

Skill: Simplify algebraic fractions by factorisation and cancellation

Question 6

Simplify $\frac{4x^2 - 9}{2x^2 - 5x + 3}$

.....

(Total 3 marks)

GCSE A* Questions

Skill: Solve a quadratic equation obtained from manipulating algebraic fractions where the variable appears in the denominator

Question 7

Solve the equation

$$\frac{7}{x+2} + \frac{1}{x-1} = 4$$

.....

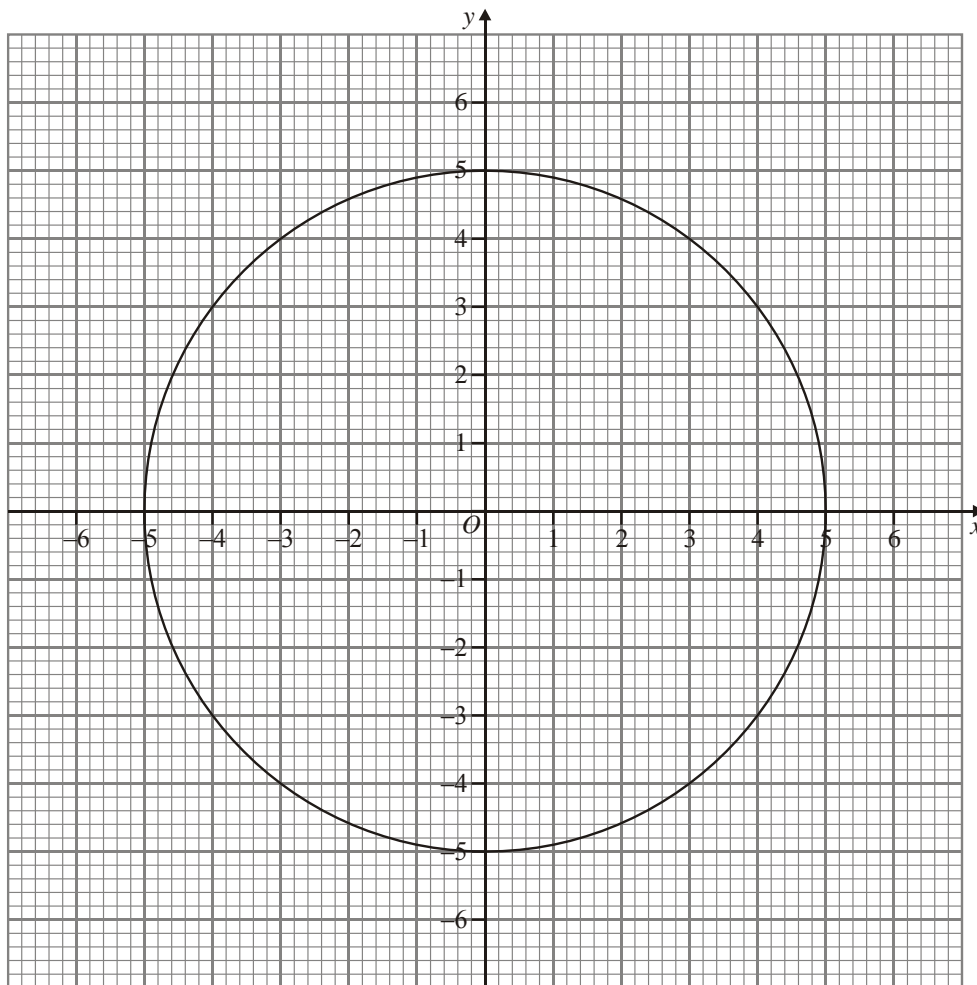
(Total 7 marks)

GCSE A* Questions

Skill: Solve equations using the intersection of two graphs

Question 8

The diagram shows a circle of radius 5 cm, centre the origin.



Draw a suitable straight line on the diagram to find estimates of the solutions to the pair of equations

$$x^2 + y^2 = 25 \text{ and } y = 2x + 1$$

$x = \dots\dots\dots, y = \dots\dots\dots$

$x = \dots\dots\dots, y = \dots\dots\dots$

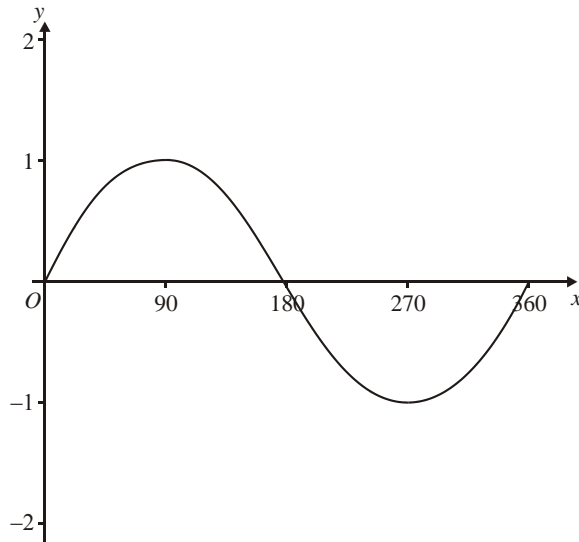
(Total 3 marks)

GCSE A* Questions

Skill: Identify the equation of a function from its graph, which has been formed by a transformation on a known function

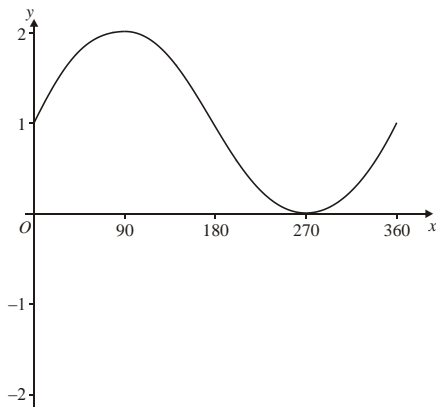
Question 9

A sketch of the curve $y = \sin x^\circ$ for $0 \leq x \leq 360$ is shown below.



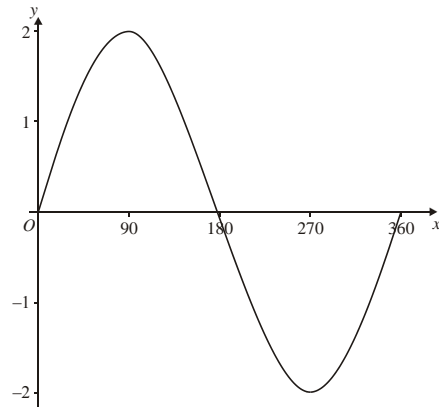
Using the sketch above, or otherwise, find the equation of each of the following two curves.

(i)



Equation $y = \dots\dots\dots$

(ii)



Equation $y = \dots\dots\dots$

(Total 2 marks)

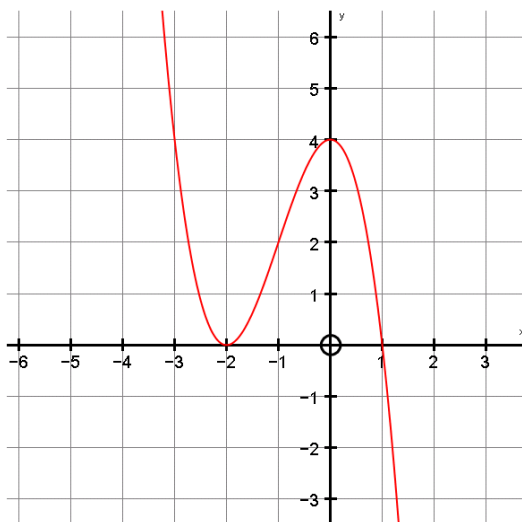
GCSE A* Questions

Skill: Transform the graph of a given function

Question 10

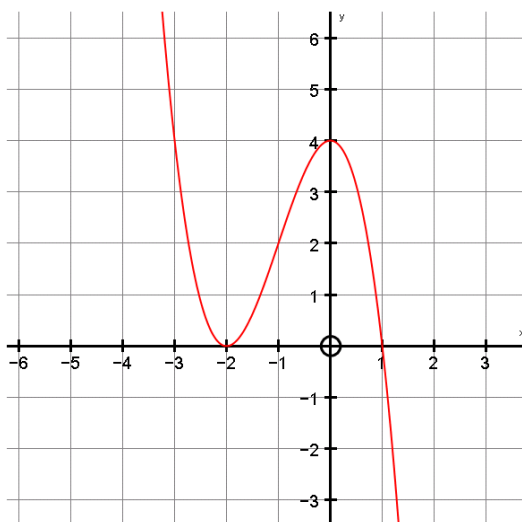
The graph of $y = f(x)$ is shown on the grids.

(a) On this grid, sketch the graph of $y = f(x) + 1$



(2)

(b) On this grid, sketch the graph of $y = f\left(\frac{x}{2}\right)$



(2)

(Total 4 marks)

GCSE A* Questions

Skill: Prove algebraic & geometric results with rigorous and logical mathematical arguments

Question 11

(a) Show that $(2a - 1)^2 - (2b - 1)^2 = 4(a - b)(a + b - 1)$

(3)

(b) Prove that the difference between the squares of any two odd numbers is a multiple of 8.

(You may assume that any odd number can be written in the form $2r - 1$, where r is an integer).

(3)

(Total 6 marks)

GCSE A* Questions

Skill: Solve real life problems that lead to constructing & solving a quadratic equation

Question 12

The diagram below shows a 6-sided shape.

All the corners are right angles.

All measurements are given in centimetres.

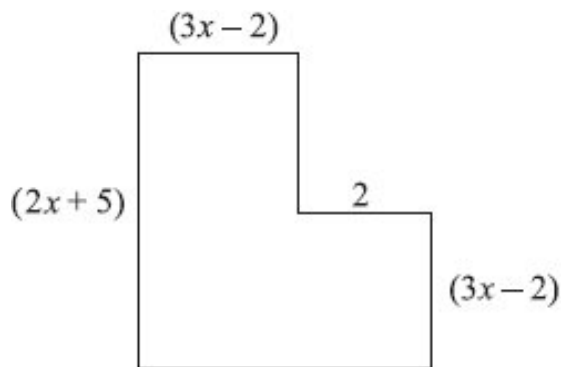


Diagram **NOT** accurately drawn

The area of the shape is 25 cm^2 .

Show that $6x^2 + 17x - 39 = 0$

(Total 3 marks)

GCSE A* Questions

Skill: Complete the square to solve problems with quadratics

Question 13

The expression $8x - x^2$ can be written in the form $p - (x - q)^2$, for all values of x .

- (a) Find the value of p and the value of q .

$$p = \dots\dots\dots$$

$$q = \dots\dots\dots$$

(3)

- (b) The expression $8x - x^2$ has a maximum value.

- (i) Find the maximum value of $8x - x^2$.

.....

- (ii) State the value of x for which this maximum value occurs.

.....

(3)

(Total 6 marks)

GCSE A* Questions

Skill: Solve 3-D problems using Pythagoras' theorem and trigonometric ratios

Question 14



The diagram represents a cuboid $ABCDEFGH$

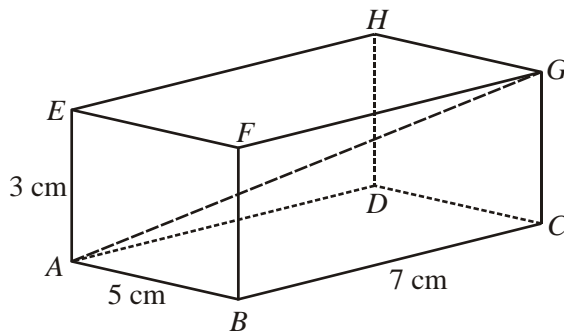


Diagram NOT accurately drawn

. $AB = 5 \text{ cm}$. $BC = 7 \text{ cm}$. $AE = 3 \text{ cm}$.

- (a) Calculate the length of AG .
Give your answer correct to 3 significant figures.

..... cm

(2)

- (b) Calculate the size of the angle between AG and the face $ABCD$.

Give your answer correct to 1 decimal place.

.....°

(2)

(Total 4 marks)

GCSE A* Questions

Skill: Solve related problems using area and volume scale factors

Question 15

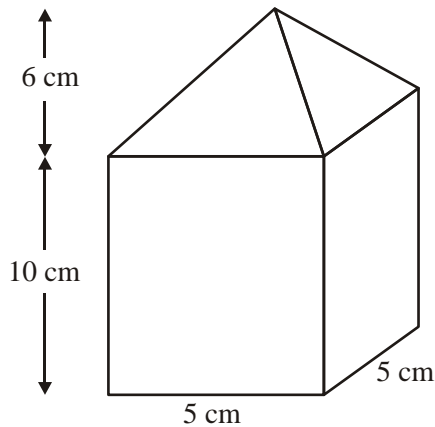


Diagram **NOT** accurately drawn

The diagram shows a model.

The model is a cuboid with a pyramid on top.

(a) Calculate the volume of the model.

..... cm³

(3)

The model represents a concrete post.

The model is built to a scale of 1:30

The surface area of the model is 290 cm².

(b) Calculate the surface area of the post.

Give your answer in square metres.

..... m²

(3)

(Total 6 marks)

GCSE A* Questions

Skill: Use circle theorems to prove geometrical results

Question 16

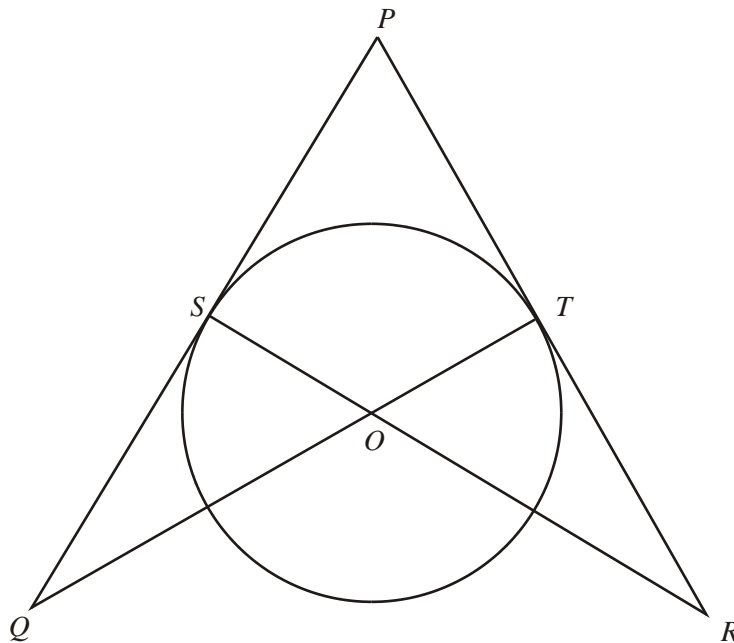


Diagram **NOT** accurately drawn

S and T are points on a circle, centre O .

PSQ and PTR are tangents to the circle.

SOR and TOQ are straight lines.

Prove that triangle PQT and triangle PRS are congruent.

(Total 3 marks)

GCSE A* Questions

Skill: Solve more complex geometrical problems using vectors

Question 17

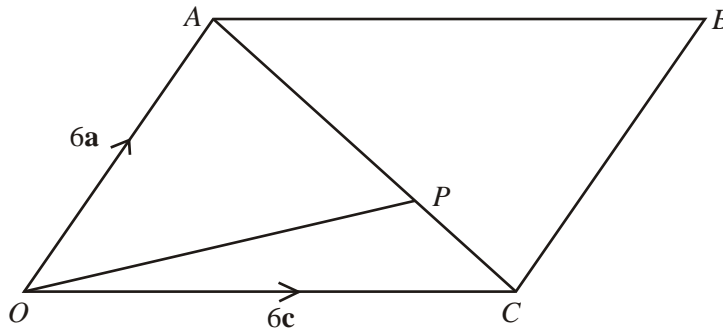


Diagram **NOT** accurately drawn

$OACB$ is a parallelogram.

P is the point on AC such that $AP = \frac{2}{3}AC$.

$$\vec{OA} = 6\mathbf{a}. \quad \vec{OC} = 6\mathbf{c}.$$

- (a) Find the vector \vec{OP} .
Give your answer in terms of \mathbf{a} and \mathbf{c} .

.....

(3)

The midpoint of CB is M .

- (b) Prove that OPM is a straight line.

(2)

(Total 5 marks)

GCSE A* Questions

Skill: Solve simple equations where the trigonometric ratio is the subject

Question 18

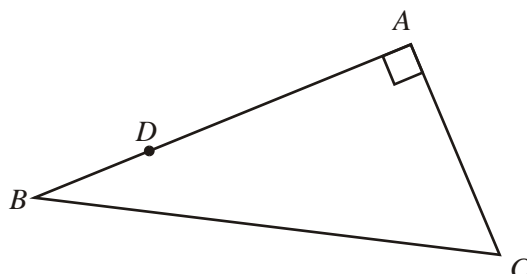


Diagram **NOT** accurately drawn

ABC is a right angled triangle.

D is the point on AB such that $AD = 3DB$.

$AC = 2DB$ and angle $A = 90^\circ$.

Show that $\sin C = \frac{k}{\sqrt{20}}$, where k is an integer.

Write down the value of k .

$k = \dots\dots\dots$

(Total 4 marks)

GCSE A* Questions

Skill: Use the cyclic properties of the graphs of sine and cosine to solve problems

Question 19

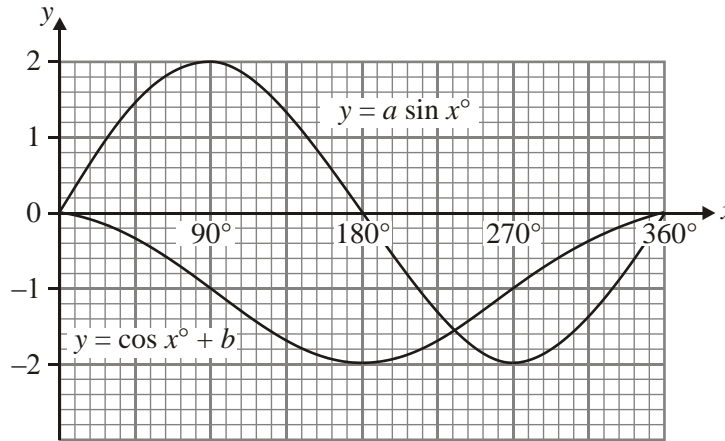


Diagram **NOT** accurately drawn

The diagram shows part of two graphs.

The equation of one graph is $y = a \sin x^\circ$

The equation of the other graph is $y = \cos x^\circ + b$

(a) Use the graphs to find the value of a and the value of b .

$a = \dots\dots\dots$

$b = \dots\dots\dots$

(2)

(b) Use the graphs to find the values of x in the range $0^\circ \leq x \leq 720^\circ$ when $a \sin x^\circ = \cos x^\circ + b$.

$x = \dots\dots\dots$

(2)

(c) Use the graphs to find the value of $a \sin x^\circ - (\cos x^\circ + b)$ when $x = 450^\circ$.

$\dots\dots\dots$

(2)

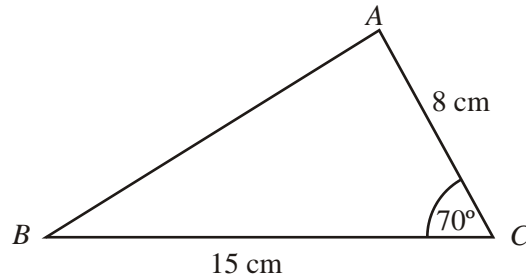
(Total 6 marks)

GCSE A* Questions

Skill: Use the sine & cosine rules to solve more complex problems involving non right-angled triangles

Question 20

Diagram **NOT** accurately drawn



In triangle ABC , $AC = 8$ cm, $BC = 15$ cm, Angle $ACB = 70^\circ$.

- (a) Calculate the length of AB .
Give your answer correct to 3 significant figures.

..... cm

(3)

- (b) Calculate the size of angle BAC .
Give your answer correct to 1 decimal place.

.....^o

(2)

(Total 5 marks)

GCSE A* Questions

Skill: solve problems involving more complex shapes & solids

Question 21

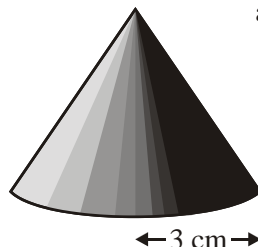
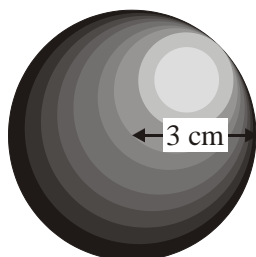


Diagram **NOT**
accurately drawn

The radius of a sphere is 3 cm.

The radius of the base of a cone is also 3 cm.

The volume of the sphere is 3 times the volume of the cone.

Work out the curved surface area of the cone.

Give your answer as a multiple of π .

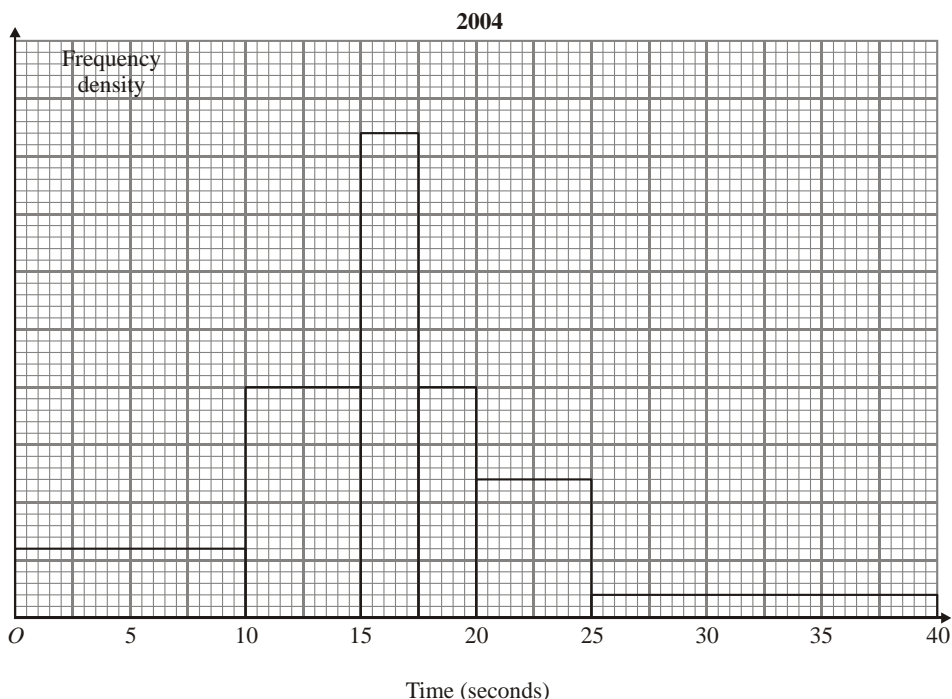
..... cm^2

(Total 7 marks)

GCSE A* Questions

Skill: Estimate statistics from a histogram

Question 22



The histogram shows information about the time it took some children to connect to the internet.

None of the children took more than 40 seconds to connect to the internet.

110 children took up to 12.5 seconds to connect to the internet.

(c) work out an estimate for the number of children who took 21 seconds or more to connect to the internet.

.....

(Total 3 marks)

GCSE A* Questions

Skill: Work out the probabilities of combined events when the probability of each event changes depending on the outcome of the previous event

Question 23

5 white socks and 3 black socks are in a drawer.



Stefan takes out two socks at random.

Work out the probability that Stefan takes out two socks of the same colour.

.....

(Total 4 marks)