4 Integration

Indefinite integrals

The rule $\int ax^n dx = \dfrac{ax^{n+1}}{n+1} + c$

can be applied to all negative or fractional values of n except $n = -1$.

For example:

Find $\int \dfrac{2}{x^2} dx$

Start by writing this as $\int 2x^{-2}\, dx$

$$= -2x^{-1} + c$$

$$= -\frac{2}{x} + c$$

N.B. NEVER FORGET THE ARBITRARY CONSTANT $+C$.

Example:

Find $\int 21x^6 - 3x^2 - \dfrac{1}{x^2} + 6\, \mathrm{d}x$.

Start by getting all $x's$ to the top $= \int 21x^6 - 3x^2 - x^{-2} + 6\, dx$

$$= \frac{21x^7}{7} - \frac{3x^3}{3} - \frac{x^{-1}}{-1} + 6x + c$$

$$= 3x^7 - x^3 + x^{-1} + 6x + c$$

Definite integrals

Questions might be asked in the form of
Example 1

Showing all your working, evaluate $\int_2^5 6x^2 + 4x \, dx$.

Integrate as per indefinite integral but because we are integrating between 2 points (5 and 2) there is no need for a constant.

$$\int_2^5 6x^2 + 4x \, dx = \left[\frac{6x^3}{3} + \frac{4x^2}{2}\right]_2^5 = \left[2x^3 + 2x^2\right]_2^5$$

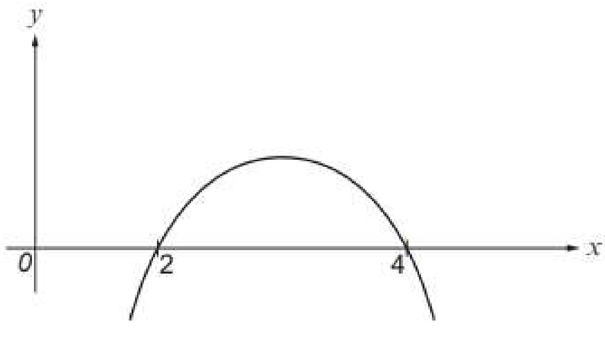
$$= (2(5)^3 + 2(5)^2) - (2(2)^3 + 2(2)^2)$$

$$= (250 + 50) - (16 + 8) \quad = 300 - 24 \quad = \mathbf{276}$$

Area under a curve

Rather than be given a definite integral as per above you may be asked to find the area under a curve (the area between the curve and the x axis)

Millie has sketched the curve $y = -x^2 + 6x - 8$.



(a) Millie states that the points (2, 0) and (4, 0) lie on the curve $y = -x^2 + 6x - 8$.
Show that Millie is correct. [2]

When $x = 2$ $\quad y = -(2)^2 + 6(2) - 8 = -4 + 12 - 8 = 0$ hence (2,0) is on the curve
When $x = 4$ $\quad y = -(4)^2 + 6(4) - 8 = -16 + 24 - 8 = 0$ hence (4,0) is on the curve

(b) Calculate the area of the region bounded by the curve $y = -x^2 + 6x - 8$ and the x-axis.
You must show all your working. [5]

The 2 points on the x axis that the curve cuts are at 2 and 4 so to find the area under the curve:-

$$\int_2^4 -x^2 + 6x - 8 \, dx = \left[\frac{-x^3}{3} + \frac{6x^2}{2} - 8x\right]_2^4 = \left(\frac{-(4)^3}{3} + 3(4)^2 - 8(4)\right) - \left(\frac{-(2)^3}{3} + 3(2)^2 - 8(2)\right)$$

$$= = \left(\frac{-64}{3} + 48 - 32\right) - \left(\frac{-8}{3} + 12 - 16\right) = -\frac{16}{3} - \left(-\frac{20}{3}\right) = \mathbf{\frac{4}{3}}$$